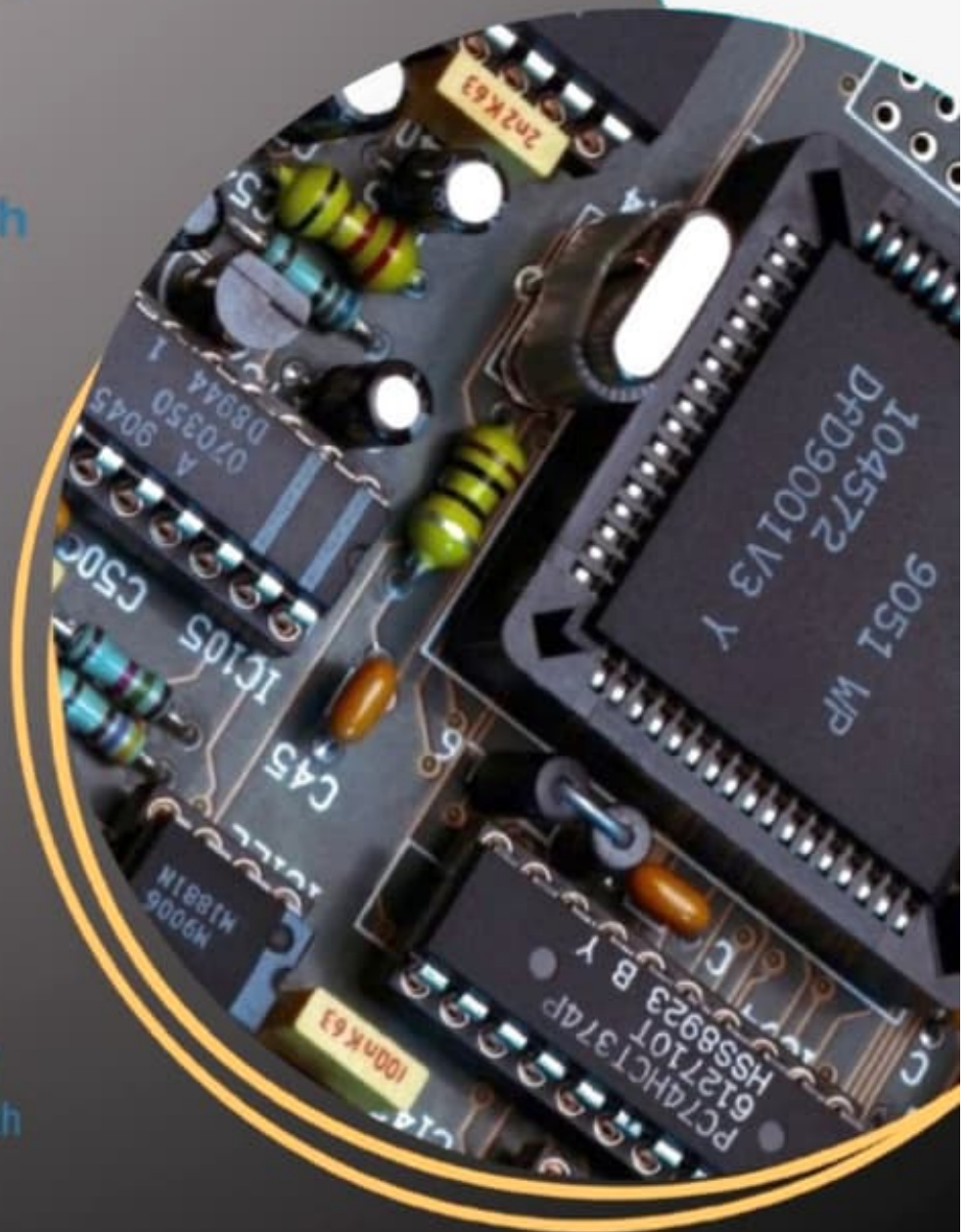


Digital ELECTRONICS

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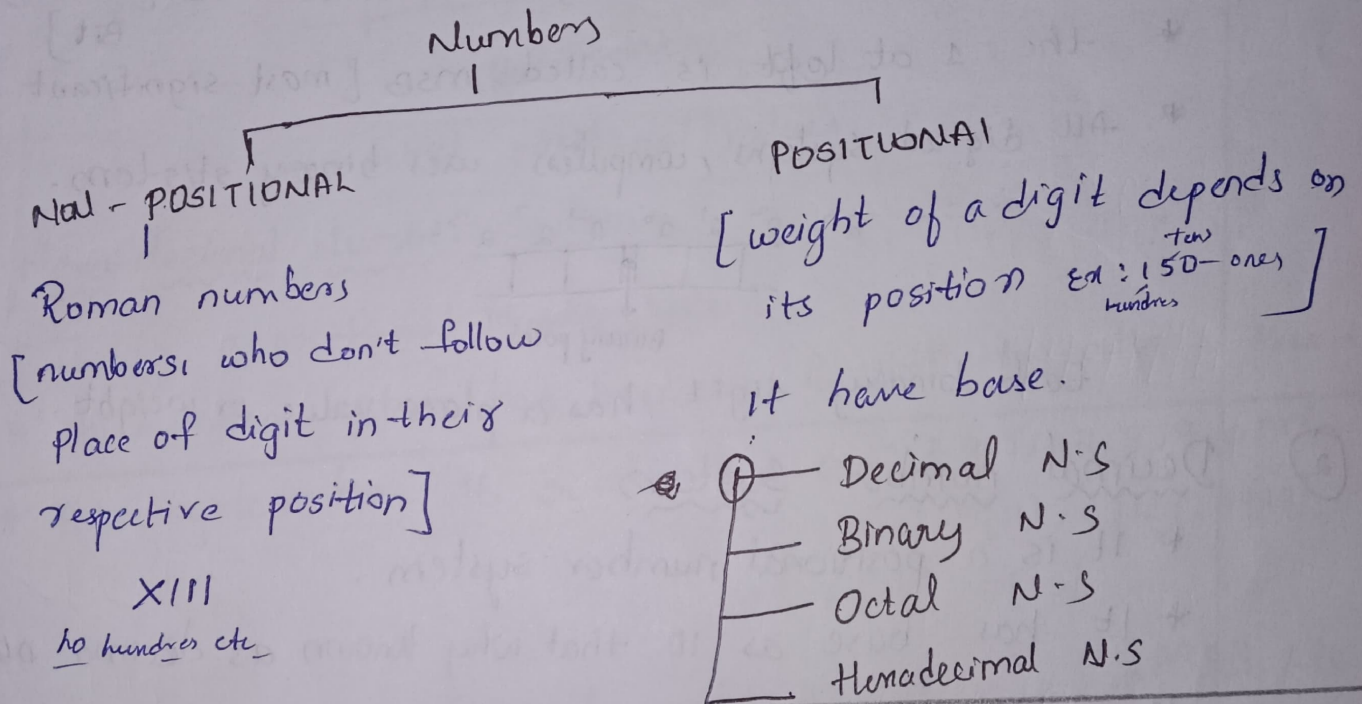
Number System

③

- ① Computers / digital systems has only 0 & 1 to represent various alpha-numeric characters.
- ② In digital systems 18 is represented as 10010 in binary.
- ③ as digits increases binary numbers also increases tremendously to decrease its length new number systems arises i.e. Octal, Hexa-decimal, Binary coded Decimal, decimal etc.

Number system.

Number = representation of Quantity.



Base

- The number of symbols can be used
- ① The no. of symbols that can be used in a number system is known as base or radix.
- ex: In binary we use only 0 & 1 so base of binary is 2
- ex: (01100110)₂

4

① Binary Number System

* It is a positioned value system.

* The number system with base 2 [only have 0 & 1 to represent anything]

* It uses only two symbols i.e 0 and 1.

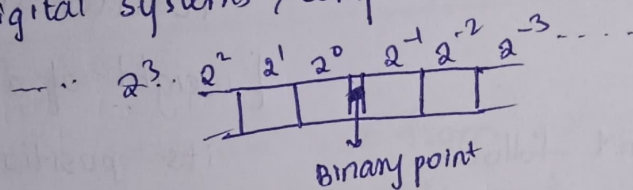
* The symbol 0 and 1 are called bit instead of digits

* Using 0 & 1 we can express ~~on~~ represent any number / character. ex: 101101.

* In 10011, the 1 at right is called LSB [least Significant Bit]

* The 1 at left is called msb [most significant Bit].

* All digital systems, computers uses binary system.



Each binary digit has a place value or weight.

② Decimal number system

* It is a positional number system.

* It has base as 10 that why known as decimal number system.

* Decimal number can have 10 different symbols

i.e 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

* They can be expressed in units, tens, hundreds, thousands

* 5678.9 can be represented as

$$5 \times 10^3 + 6 \times 10^2 + 7 \times 10^1 + 8 \times 10^0 + 9 \times 10^{-1}$$

$$5000 + 600 + 70 + 8 + 0.9 = 5678.9$$

* 5 is left has greatest weight so msb or msd

* 9 is right has lowest weight so LSB or lsd digit

Octal Number System

* It is a positional N.S.

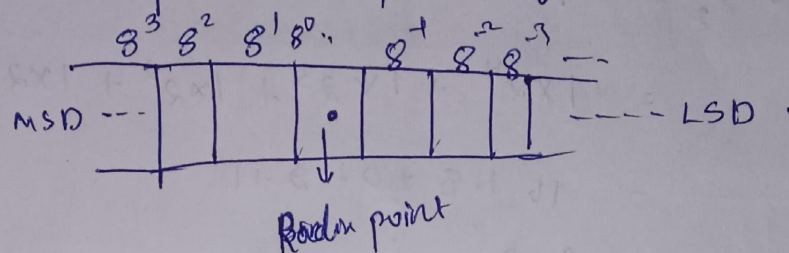
* The number system with base 8 is called octal N.S.

* This system has eight symbols i.e 0, 1, 2, 3, 4, 5, 6 and 7

Ex: 1703

* Used in many microcomputers & computers for entering data as 3-bit binary data.

* weight is expressed as power of 8.



octal positional values

Hexa-decimal Number System

* It is a positional Number system.

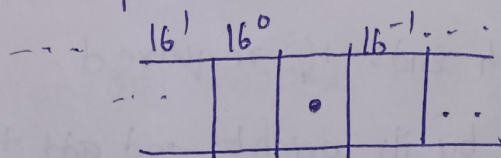
* The number system has base as 16, that value called as Hexa-d

* This system has 16 symbols i.e 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 and A [10], B [11], C [12], D [13], E [14], F [15].

* It is a combination of numbers & alphabets as 2ABD3, etc.

* widely used for representing microcomputer memory location addresses.

* weight is represented as power 16



⑥

NUMBER SYSTEMS CONVERSIONS

① BINARY TO DECIMAL.

- ① Write down all bits in a row
- ② Multiply each bit with its positional weight
- ③ Add all ~~bits~~ of them.

② Convert binary no' $(11011)_2$ into decimal.

$$\begin{aligned}
 \text{A) } (11011)_2 &= \overset{4}{1} \overset{3}{1} \overset{2}{0} \overset{1}{1} \overset{0}{1} \\
 &= 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\
 &= 16 + 8 + 0 + 2 + 1 \\
 &= (27)_{10}.
 \end{aligned}$$

② $(1101.101)_2$ to decimal.

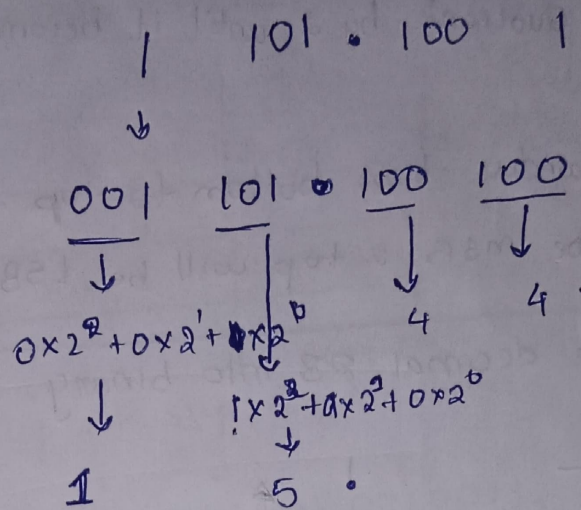
$$\begin{aligned}
 N &= 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3} \\
 &= 8 + 4 + 0 + 1 + 0.5 + 0 + 0.125 \\
 &= (13.625)_{10}.
 \end{aligned}$$

② BINARY TO OCTAL.

- ① Arrange the bits in a set of three bit. from right to left & left to right after decimal point
- ② Add 0's at both ends if required.
- ③ multiply each bit by its weight and add them do this for all parts separately.
- ④ Then write its octal equivalent

Convert 1101.1001 into octal.

Sol Step ①

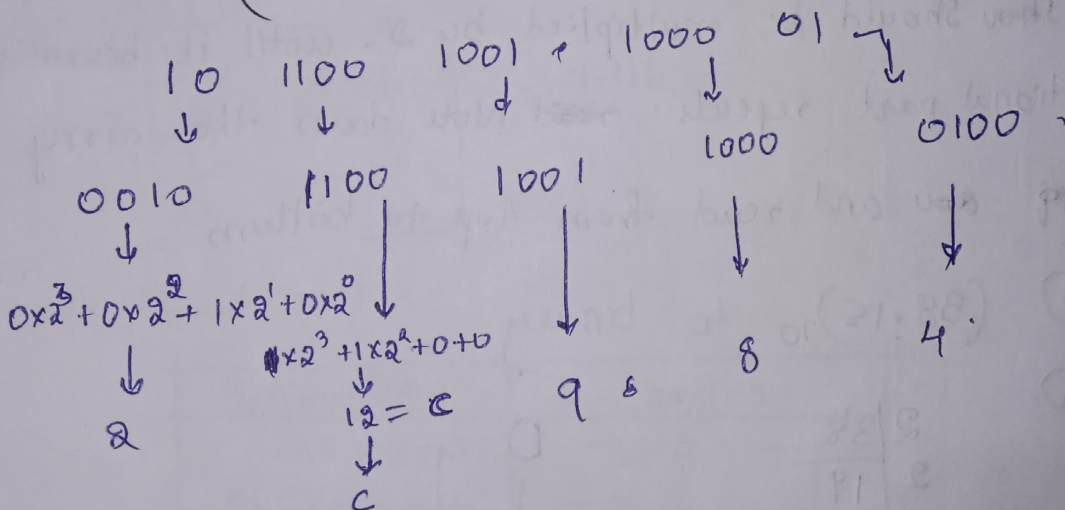


$$\therefore 1101.1001 = (15.44)_8$$

Binary to Hexadecimal

* Do same procedure but now create 4 pairs

convert $(1011001001.100001)_2$ into hexadecimal.



$$\therefore (1011001001.100001)_2 = (2C9.84)_{16}$$

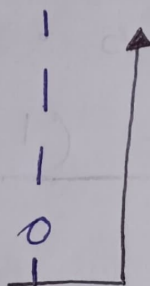
⑧

① DECIMAL TO OTHERS DECIMAL TO BINARY

- ① Divide by 2
- ② Note down remainder after each division.
- ③ Divide the quotient by 2 until it becomes zero & note the remainder.
- ④ ~~Write~~ write remainders from bottom to top such that bottom will be MSB & top will be LSB.

① Convert the decimal 23 into binary.

2	23
2	11
2	5
2	2
2	1
0	



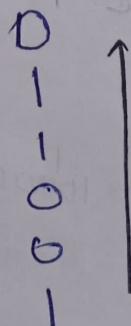
(Down to up) $(23)_{10} = (10111)_2$

* If the decimal contains fraction then the fractional part ~~shd~~ should be multiplied by 2, until it becomes 0 or fractional part repeats. ~~and~~ Note down the carry part ~~if~~ and read from top to bottom

Example ② $(38.15)_{10}$ to binary

Step ①

2	38
2	19
2	9
2	4
2	2
2	1
0	



$\Rightarrow 100110$

Step ② - fractional part.

$\times 2$	Carries
$0.15 \times 2 = 0.3$	$\rightarrow 0$
$0.3 \times 2 = 0.6$	$\rightarrow 0$
$0.6 \times 2 = 1.2$ ^{take 0.2}	$\rightarrow 1$
$0.2 \times 2 = 0.4$	$\rightarrow 0$
$0.4 \times 2 = 0.8$	$\rightarrow 0$
$0.8 \times 2 = 1.6$	$\rightarrow 1$
	continues

0
0
1
0
0
1

$= 0.001001$

So add step 1 & step 2 results

$$(38.15)_{10} = (100110.001001)_2$$

② Decimal to Octal

* Same process but divide by 8 & for fractional multiply by 8
convert $(974.35)_{10}$ in octal.

$$\begin{array}{r|l} 8 & 974 \\ \hline 8 & 121 \\ 8 & 15 \\ 8 & 1 \\ & 0 \end{array} \quad \begin{array}{r} - 6 \\ - 1 \\ - 7 \\ - 1 \end{array} \quad \begin{array}{l} \uparrow \\ \uparrow \\ \uparrow \\ \uparrow \end{array} = 1716$$

fractional part

Fractional $\times 8$	Carries
$0.35 \times 8 = 2.80$	$\rightarrow 2$
$0.8 \times 8 = 6.4$	$\rightarrow 6$
$0.4 \times 8 = 3.2$	$\rightarrow 3$
$0.2 \times 8 = 1.6$	$\rightarrow 1$
$0.6 \times 8 = 4.8$	$\rightarrow 4$

$= 0.26314$

$$(974.35)_{10} = (1716.26314)_8$$

(10)

③ DECIMAL TO HEXADECIMAL CONVERSION

same procedure but divide by 16 and ^{for} fractional part multiply by 16.

Convert $(28.6875)_{10}$ into hexadecimal.

$$\begin{array}{r} 16 \overline{) 28} \\ 16 \overline{) 1} \\ \hline 0 \end{array} \quad \begin{array}{l} - 12 = C \\ - 1 \end{array} \quad \begin{array}{c} C \\ 1 \end{array} \uparrow = 1C$$

$$0.6875 \times 16 = 11.0 \rightarrow B$$

$$\therefore (28.6875)_{10} = 1C.B$$

① OCTAL TO BINARY

~~Octal~~ ① Octal can be converted to binary by writing

each bit with its 3-bit equivalent binary.

Convert $(23.54)_8$ into binary

$$\begin{array}{ccc} 2 & 3 & . & 5 & 4 \\ \downarrow & \downarrow & & \downarrow & \downarrow \\ 010 & 011 & & 101 & 100 \end{array}$$

$$(23.54)_8 = (10011101100)_2$$

② Octal to decimal

Similar to conversion of binary to decimal but only difference is that the power of 8 is used instead of 2.

Convert $(567)_8$ into decimal

$$\begin{array}{ccc} 5 & 6 & 7 \\ \swarrow & | & \searrow \\ 5 \times 8^2 & + 6 \times 8^1 & + 7 \times 8^0 \end{array}$$

$$= 320 + 48 + 7$$

$$= (375)_{10}$$

② 22.78_8

$$= 2 \times 8^1 + 2 \times 8^0 + 7 \times 8^{-1} + 8 \times 8^{-2}$$

$$= 16 + 2 + 0.875 + 0.0625$$

$$= (18.9375)_{10}$$

③ Octal to Hexa-decimal

Step - ① do first octal to binary

Step - ② Then Binary to hexadecimal

Convert $(615)_8$ to hexadecimal

Step - ①

$$\begin{array}{ccc} 6 & 1 & 5 \\ \downarrow & \downarrow & \downarrow \\ 110 & 001 & 101 \end{array}$$

$$\text{Binary number} = (110001101)_2$$

Step - ②

$$\begin{array}{ccc} 0001 & 1000 & 1101 \\ \downarrow & \downarrow & \downarrow \\ 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 & & 13 \\ \downarrow & \downarrow & \downarrow \\ 1 & 8 & D \end{array}$$

$$\therefore \text{Hexadecimal number} = (18D)_{16}$$

(12)

① Hexadecimal to BINARY

Convert $(2BD)_{16}$ to binary.

~~Step~~ ① Convert each bit into 4 bit equivalent binary by division method.

2	B	D
↓	↓	↓
11	11	13
↓	↓	↓
0010	1011	1101

$$(2BD)_{16} = (1010111101)_2.$$

2 13	1
2 6	0
2 3	1
2 1	1
0	↑
2 11	1
2 5	1
2 2	0
2 1	1
0	↑
2 2	0
2 1	1
0	↑

② Hexadecimal to decimal

~~Convert~~ * same as conversion of binary/octal to decimal.

$$(5D.98)_{16} \quad D=13$$

$$= 5 \times 16^1 + 13 \times 16^0 + 9 \times 16^{-1} + 8 \times 16^{-2}$$

$$= 80 + 13 + 9/16 + 8/256$$

$$= 80 + 13 + 0.5625 + 0.03125$$

$$= 93.59375$$

$$(5D.98)_{16} = (93.59375)_{10}.$$

② Hexadecimal to Octal number system

Same as conversion of octal to Hexadecimal

Convert $(B4)_{16}$ into hexadecimal.

Step ① : Convert to Binary.

$$\begin{array}{cc}
 B & 4 \\
 \downarrow & \downarrow \\
 11 & 4 \\
 \downarrow & \downarrow \\
 1011 & 0100
 \end{array}$$

$$\therefore \text{Binary} = (10110100)_2$$

~~Convert~~
Step ② : Binary to ~~Hexa~~-octal:

$$\begin{array}{ccc}
 010 & 110 & 100 \\
 \downarrow & \downarrow & \downarrow \\
 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 & & \\
 \downarrow & & \\
 2 & 6 & 4
 \end{array}$$

$$\therefore (BA)_{16} = (264)_8$$

BINARY ARITHMETIC

* Digital systems deal with binary numbers hence the arithmetic operation is also performed logically.

BINARY ADDITION

A+B BINARY	Sum	Carries
0+0	0	0
0+1	1	0
1+0	1	0
1+1	0	1

* At last $1+1$ gives 2 but in binary it is 10 in which LSB '0' given as sum and MSB '1' is given as carry.

* Add 1010 & 1011 & verify decimal.

Binary Addition.

$$\begin{array}{r}
 1010 \\
 + 1011 \\
 \hline
 10101
 \end{array}$$

Verify $\rightarrow$

$$\begin{array}{r}
 1010 \rightarrow 10 \\
 1011 \rightarrow 11 \\
 \hline
 21
 \end{array}$$

* Add 1011.011 & 110.1

$$\begin{array}{r}
 1011.011 \\
 + 110.100 \\
 \hline
 10001.111
 \end{array}$$

$$\begin{array}{r}
 11.375 \\
 6.5 \\
 \hline
 17.875
 \end{array}$$

BINARY SUBTRACTION

A-B	Difference	Borrow
0-0	0	0
0-1	1	1
1-0	1	0
1-1	0	0

Perform Subtraction 10101 from 11011

$$\begin{array}{r}
 \textcircled{1} \\
 11011 = (27)_{10} \\
 - 10101 = (21)_{10} \\
 \hline
 00110 = (6)_{10}
 \end{array}$$

BINARY MULTIPLICATION

* Easier than decimal multiplication as has only 0 & 1 but same

$$A \times B = 0$$

$$0 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 0 = 0$$

$$1 \times 1 = 1$$

multiply 1011 by 101

$$\begin{array}{r}
 1011 \\
 \times 101 \\
 \hline
 1011 \\
 0000 \\
 1011 \\
 \hline
 11011
 \end{array}$$

$$\begin{array}{r}
 \text{Verification} \\
 11 \times 5 \\
 \hline
 = 55
 \end{array}$$

BINARY Complement

Used to represent negative numbers.

They are basically two complements i.e. 1's complement & 2's complement.

1's complement

It is the complement of each bit

The complement of 0 is 1 and complement of 1 is 0.

Ex: 1110101 [Find 1's complement]

1's of 1110101 is 0001010

2's complement

If we add '1' to 1's complement then it is called 2's complement

$$2's \text{ complement} = 1's \text{ complement} + 1$$

Find 2's complement of 1001 (1001)₂

$$\text{number} = 1001$$

$$1's = 0110$$

$$2's = \begin{array}{r} 0110 \\ + 1 \\ \hline 0111 \end{array}$$

(18)

I SUBTRACTION OF smaller number from larger using complement method.

~~Ex~~ Ex

* Find 1's complement of given ^{smaller} number

* Add the 1's complement to larger number

* remove carry & Add it to the result known as end-around carry.

Exr $101011 - 111001$ using 1's method

$$\begin{array}{r} \cancel{101011} \\ - \cancel{111001} \\ \hline 0 \end{array}$$

1's of 101011 is 000100

$$\begin{array}{r} 111001 \\ 010100 \\ \hline \end{array}$$

$$\begin{array}{r} 1001101 \\ \hline \end{array}$$

← Add end carry

$$\begin{array}{r} 1110 \\ \hline \end{array}$$

← result

II

Subtraction of larger number from smaller number using 1's complement method.

- * Find 1's of larger number
- * Add 1's complement larger to smaller
- * Their result is negative ~~put~~ & in 1's form.
- * Again do complement for final answer & put (-) sign.

Subtract 1100 from 1000 using 1's complement

- * 1's complement of ^{larger}~~smaller~~ number 1100 is 0011

*

$$\begin{array}{r} 0011 \\ + 1000 \\ \hline \end{array}$$

1011 ← addition of 1's of larger to smaller

* True result = 0100 = -0100.

2's complement method.

Subtraction of smaller from larger.

- ① Find 2's of ~~large~~ smaller
- ② Add it to larger
- ③ Discard the carry.

Ex 101.11 from 1100.1 by 2's c.m

smaller = 0101.11 (Add precise zero)

1's of smaller = 1010.00

2's = 1010.01

$$\begin{array}{r} 1100.10 \\ + 1010.01 \\ \hline 10110.11 \\ \hline \end{array}$$

True result = 110.11.

Subtraction of larger from smaller using 2's cm.

- * Find 2's of larger number
- * Add 2's to smaller number
- * result is negative & in 2's complement form.
- * To get actual result take 2's complement the result.

Ex: $1101 - 1000$

* 1's of 1000 is ~~1111~~ 0010

2's of 1000 is 0011

$$\begin{array}{r} \text{smaller num is } 1000 \\ + \quad 0011 \\ \hline 1011 \end{array}$$

1011 is in 2's form again write 2's form of 1011

2's of result will be $1011 \rightarrow 0100 + 1 = 0101$

∴ final answer = 0101

Boolean Algebra: It is defined with set of Elements ^{binary variable} a set of operators & a number of postulates. (21)

Boolean Algebra

- * Boolean Algebra invented by George Boolean.
- * in boolean algebra every variable can have only 2 values ^{TRUE/FALSE} 1/0
- * the digital signals are discrete in nature and can only assume 0's.
- * The TRUE/FALSE logic of Boolean algebra is the basis of all digital systems like computers, calculators, etc.
- * Binary variables can be represented by A, B, C, ... & their value can have only 0 or 1.

Postulates in BOOLEAN ALGEBRA

- * Postulates: Any-thing which is not proved but assumed to be true.
- * Variables (binary) or Elements ~~are~~ are actually input signals of the digital circuit.
- * variables represented by letters for analysing & designing of digital systems.
- * Three basic types of operators are available i.e. AND (.), OR (+), and NOT (bar).

6 Postulates of Boolean Algebra

- ① The result of boolean algebra is either 0 or 1.
for example, if $A=0$ & $B=1$, then $A+B=1$, $A \cdot B=0$.
- ② They are two elements 0 & 1
0 is added to binary variable A [result = A] i.e. $A+0=A$
1 is multiplied to binary variable A [result = A] i.e. $A \cdot 1=A$
- ③ Binary elements follows commutative law
 $A+B = B+A$
 $A \cdot B = B \cdot A$
- ④ Binary elements follows distributive law
 $A+(B \cdot C) = (A+B) \cdot (A+C)$
 $A \cdot (B+C) = A \cdot B + A \cdot C$
- ⑤ The complement of A is such that if $A=1$ then $\bar{A}=0$
if $A=0$ then $\bar{A}=1$ i.e. $A+\bar{A}=1$ & $A \cdot \bar{A}=0$
- ⑥ There are atleast two elements [Boolean] A & B , where
 $A, B \in C$ such that $A \neq B$.

Laws of Boolean Algebra

①

OR laws

$$(i) A+0=A$$

$$(ii) A+A=A$$

$$(iii) A+1=1$$

$$(iv) A+\bar{A}=1$$

let $A=0$ then	$0+0=0$	let $A=1$ then	$1+0=1$
	$0+0=0$		$1+1=1$
	$0+1=1$		$1+1=1$
	$0+1=1$		$1+0=1$

②

AND laws

$$(i) A \cdot 0=0$$

$$(ii) A \cdot 1=A$$

$$(iii) A \cdot A=A$$

$$(iv) A \cdot \bar{A}=0$$

NOT laws

$$(i) \bar{0}=1$$

$$(ii) \bar{1}=0$$

$$(iii) \text{If } A=0, \text{ then } \bar{A}=1$$

$$(iv) \text{If } A=1, \text{ then } \bar{A}=0$$

$$(v) \bar{\bar{A}}=A$$

④

commutative laws

$$(i) A+B=B+A$$

$$(ii) A \cdot B=B \cdot A$$

Associative laws

$$(i) A+(B+C)=(A+B)+C$$

$$A \cdot (B \cdot C)=(A \cdot B) \cdot C$$

⑥

Distributive laws

$$(i) A \cdot (B+C)=A \cdot B + A \cdot C$$

$$(ii) A+(B \cdot C)=(A+B) \cdot (A+C)$$

$$(iii) A+\bar{A}B=A+B$$

Auxiliary law

$$(i) A+\bar{A}B=A+B$$

$$(ii) A+A \cdot B=A$$

De-morgan's law

$$(i) \overline{A+B}=\bar{A} \cdot \bar{B}$$

$$(ii) \overline{A \cdot B}=\bar{A} + \bar{B}$$

DEMORGAN'S THEOREM

They are two laws of demorgan

I Theorem - 1

1st theorem states that the complement of sum of ~~two~~ Equal the product of the complements.

$$\overline{A+B} = \bar{A} \cdot \bar{B}$$

Proof by truth table

LHS		Result	
A	B	A+B	$\overline{A+B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

RHS		Result		
A	B	$\bar{A}$	$\bar{B}$	$\bar{A} \cdot \bar{B}$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	0	0	0

As the result of both LHS & RHS is equal i.e. 1,0,0,0 so
 $\overline{A+B} = \bar{A} \cdot \bar{B}$

II Theorem - 2

The second theorem says that the complement of product is equals to the sum of complements.

$$\overline{AB} = \bar{A} + \bar{B}$$

Proof

A	B	AB	(Result) $\overline{AB}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

A	B	$\bar{A}$	$\bar{B}$	($\bar{A} + \bar{B}$ Result)
0	0	1	1	1
0	1	1	0	1
1	0	0	1	1
1	1	0	0	0

Result of LHS = RHS $\therefore \overline{AB} = \bar{A} + \bar{B}$

Logic gates: Logic gate is defined as the basic logic circuit used to perform logical operation such as AND, OR, NOT etc.

* Logic gate is electronic circuit

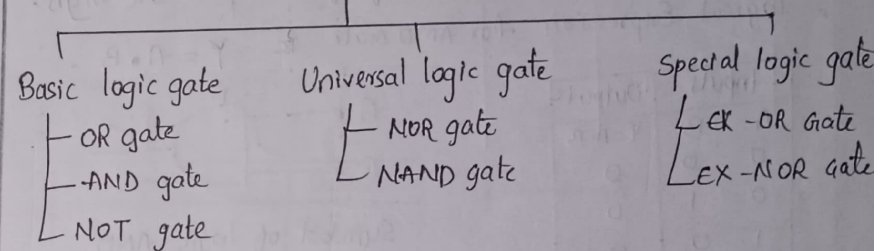
* Operation of logic gates can be examined by truth table

* It takes ~~one~~ one/more inputs & generates one output

Truth table: Shows all the input-output possibilities of a logic circuit

Classification of logic gates

Logic gates

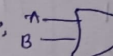


Basic logic gate: They are of three types

(i) OR gate

- OR gate performs logical addition.
- It requires two inputs or more ^(A & B) and gives one output.
- An OR gate is a gate whose output is ^{low} high if any or all of the inputs are ~~high~~ low

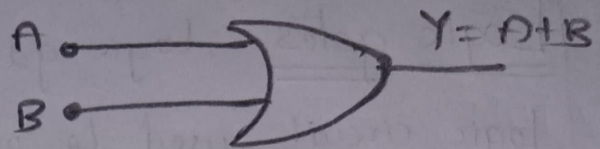
Logical expression for OR gate is $Y = A+B$

Symbol: 

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Expression $Y = A + B$

Inputs		Output
A	B	$Y = A + B$
0	0	0
0	1	1
1	0	1
1	1	1



Symbol of logic OR

2 AND Gate

- AND Gate performs logical multiplication.
- It is a logic gate which require two/more inputs for one output (Y)
- It is output high only if all inputs are high.
- Logical Expression for AND Gate is $Y = A \cdot B$

Inputs		Output
A	B	$Y = A \cdot B$
0	0	0
0	1	0
1	0	0
1	1	1

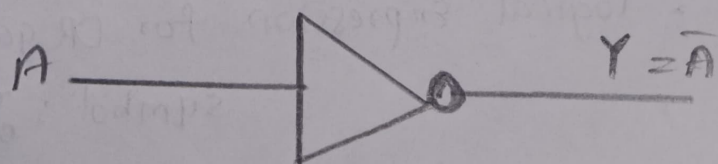


Symbol of logic AND

3 NOT Gate

- It requires only one input whose output is always complement of input also known as inverter
- It shows the output high if input is low & vice versa.
- Expression is $Y = \bar{A}$

Inputs	Outputs
A	$Y = \bar{A}$
0	1
1	0



Symbol of logic NOT

Universal logic Gates

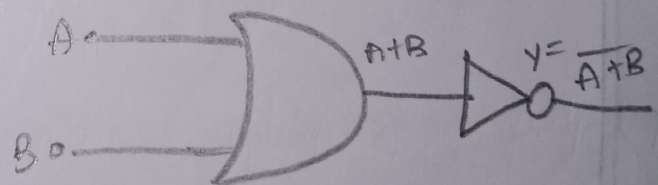
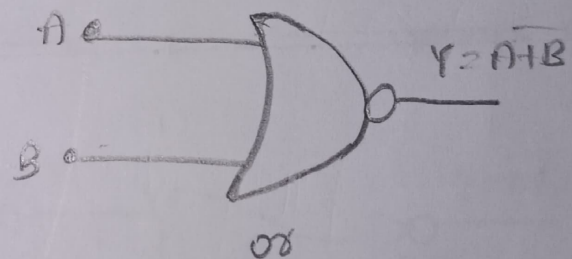
* These logic gates formed by combination of two basic logic gates

* They are of two types

① NOR Gates

- This gate is the combination of ~~two~~ NOT & OR gate.
- This NOR gate requires two / more inputs for one output
- The output of NOR gate is high only when all the inputs are low. If any one or both inputs are high then output will be low.
- Logical Expression is $Y = \overline{A+B}$
- output = complement of OR.

Inputs		Output
A	B	$Y = \overline{A+B}$
0	0	1
0	1	0
1	0	0
1	1	0

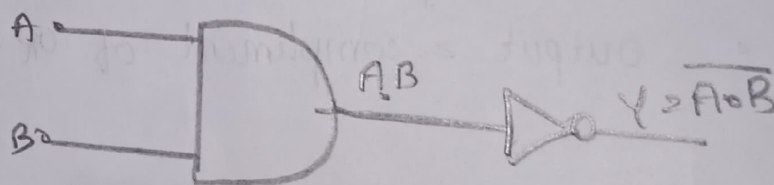
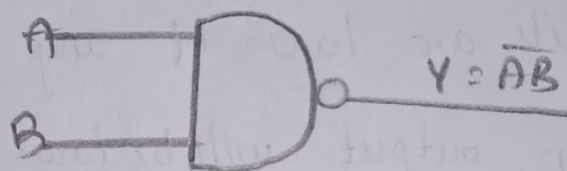


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② NAND Gate

- This gate is formed by the combination of NOT & AND gate
- This " requires two or more inputs but one output
- The output of NAND gate is low when all the inputs are high. If anyone or all inputs are low, then the output of NAND gate will be high.
- Logical expression for NAND gate is $Y = \overline{A \cdot B}$

Inputs		Output
A	B	$Y = \overline{A \cdot B}$
0	0	1
0	1	1
1	0	1
1	1	0



Ex-NOR (Gate)

* An Ex-NOR gate has two inputs but only one output (Y)

* The output of Ex-NOR (gate) is HIGH if even number of inputs are 1.

* The output of Ex-NOR gate is LOW if odd no. of inputs are 1.

* Logical Expression is $Y = \overline{A \oplus B} = AB + \overline{A}\overline{B}$

Derived as $\overline{A \oplus B} = \overline{AB + \overline{A}\overline{B}}$

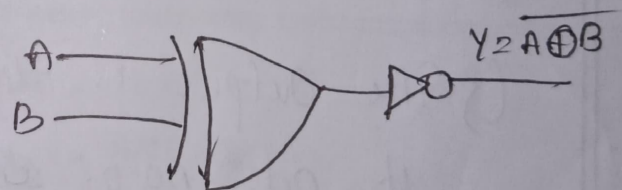
$$= \overline{AB} + \overline{\overline{A}\overline{B}}$$

$$= (\overline{A} + \overline{B})(A + B)$$

$$= AB + \overline{A}\overline{B}$$

Truth Table

Inputs		Output
A	B	$Y = \overline{A \oplus B}$
0	0	1
0	1	0
1	0	0
1	1	1



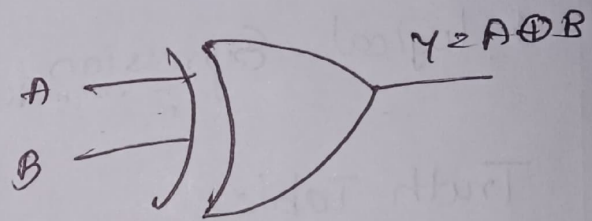
→ Used as bit comparator as if inputs both inputs are 0 or 1 then only the result is HIGH.

EX-OR gate

- EX-OR can be called as Exclusive OR gate
- EX-OR gate has two/more inputs but one output
- The output of EX OR gate is high if two inputs are different
- The output of EX OR gate is low if all the inputs are either low or high.

Expression $Y = A \cdot B = \bar{A}B + A\bar{B}$

Input		Outputs
A	B	$Y = A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

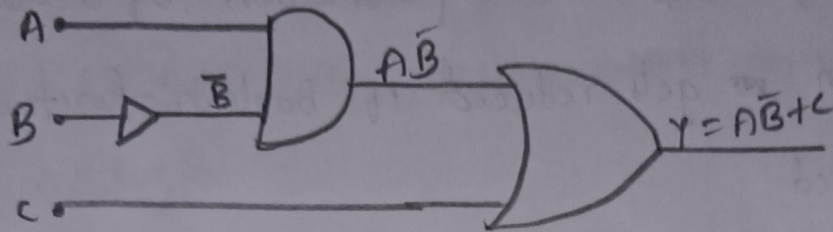


(c) The output is High
if odd no. of inputs
are 1

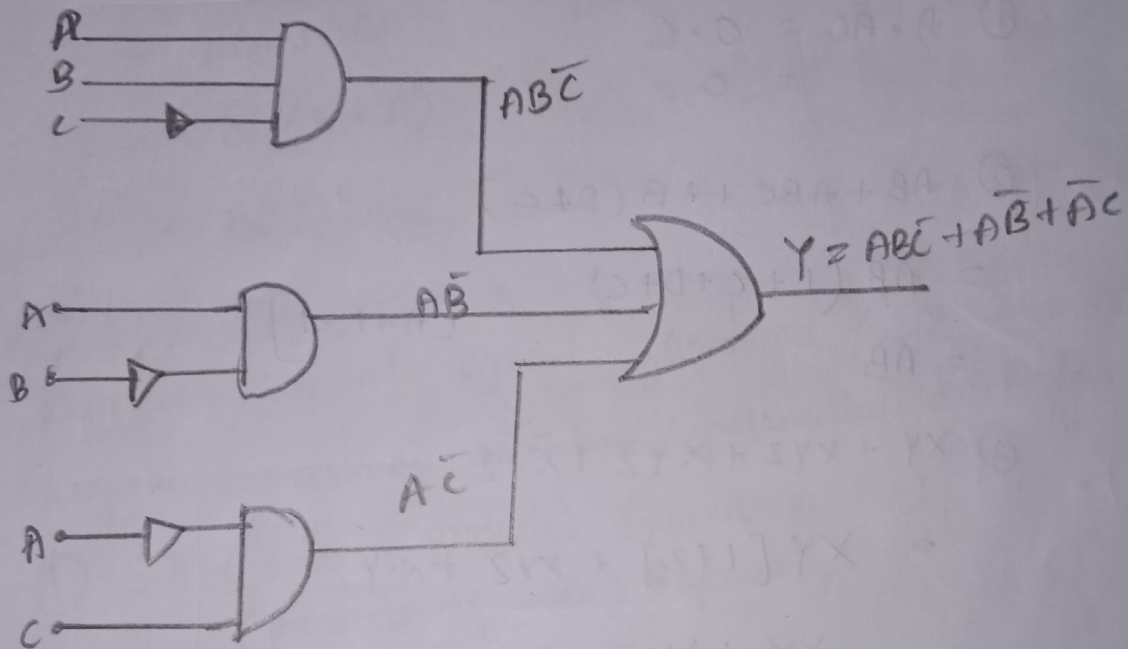
Realisation using NAND Gate

$$Y = A\bar{B} + C$$

A, B are literals



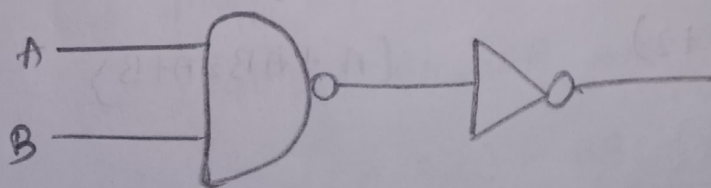
$$Y = ABC + A\bar{B} + A\bar{C}$$



AND GATE USING NAND GATE

Complement of NAND GATE is AND GATE.

$$\overline{\overline{A \cdot B}} = A \cdot B$$



Simplification of Boolean Expression

* the no. of gates required & the number of input terminals for the gate for the implementation of a boolean function gets reduced if boolean function is simplified

* $\therefore$ Reduce complexity of hardware.

Examples

$$\textcircled{1} A \cdot \bar{A}C = 0 \cdot C \\ = 0.$$

$$\textcircled{2} AB + ABC + AB(D+E) \\ = AB(1 + C + D + E) \quad [A+1=1] \\ = AB$$

$$\textcircled{3} XY + XYZ + XY\bar{Z} + \bar{X}YZ \\ = XY[1+Z] + XY\bar{Z} + \bar{X}YZ \\ = XY + XY\bar{Z} + \bar{X}YZ \quad [A+1=1] \\ = XY(1+\bar{Z}) + \bar{X}YZ \\ = XY + \bar{X}YZ \quad [A+1=1] \\ = Y(X + \bar{X}Z) \\ = Y(X+Z) \quad [A + \bar{A}B = A+B]$$

$$④ \quad \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC$$

$$\bar{A}\bar{C} \cdot (\bar{B} + B) + \bar{A}BC$$

$$\bar{A}\bar{C} + \bar{A}BC \quad (A + \bar{A} = 1)$$

$$\bar{A}(\bar{C} + BC) \quad [A + \bar{A}B = A + B]$$

$$\bar{A}(\bar{C} + B)$$

$$⑤ \quad \bar{A}B\bar{C}D + \bar{A}BCD + ABD$$

$$= \bar{A}BD(\bar{C} + C) + ABD$$

$$= \bar{A}BD + ABD$$

$$= BD(\bar{A} + A)$$

$$= BD(1)$$

$$= BD$$

Representation of logical functions (SOP AND POS)
 Logical Expressions can be represented in 2 forms

① SOP (sum of products) Expressions

② POS (products of sum) Expressions.

SOP Expressions

• SOP Expressions contains two or more AND functions and OR functions together

② • These products are called as minterms

③ • Examples : of SOP are as follows

$$(i) \quad f(A, B, C) = \overbrace{ABC + ABC}^{\text{sum}}$$

↓
product terms

① * definition : SOP expressions is a set of AND (product) terms that are summed (ORed) together

① POS Expression [POS Expressions are the set of OR (sum) terms which are ANDed (product) together.]
 X. The product of sums expression contains two or more OR functions & AND together.

②. These products are called minterms.

③. The product of sum is a dual of sum of product

$$S_{xy} = (\bar{A} + \bar{B} + C) \cdot (A + B + C) \cdot (A + B + C) \rightarrow$$

└───┘
└───┘
 sum terms product

Standard form of boolean expression

①. If each product term in SOP form contains all the variables then the SOP form is known as Standard or canonical SOP form.

②. If each sum term in POS form contains all the variables then the POS form is known as Standard / canonical POS form.

converting SOP to canonical form.

① Identify missing variable

② AND (multiply) each product term having missing variables with the ORing (adding) the missing variable & its complement.

③ Expand equation by applying distributive law

Example:

$$f = A + AB + ABC$$

$$= A(B+B)(C+C) + AB(C+C) + ABC$$

$$= (AB + AB)(C+C) + ABC + ABC + ABC$$

$$= \cancel{ABC} + \cancel{AB\bar{C}} + \cancel{A\bar{B}C} + \cancel{AB\bar{C}} + \cancel{A\bar{B}C}$$

$$= \cancel{ABC} + A$$

$$= ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + \cancel{ABC} + \cancel{AB\bar{C}} + \cancel{A\bar{B}C}$$

$$= ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C}$$

$$= m_7 + m_6 + m_5 + m_4 = \sum (4, 5, 6, 7)$$

Steps to convert POS to canonical form

- ① Identify the missing variable in each sum term if any
- ② OR each sum term having missing variables with term form by ANDing the variable and its complement.
- ③ Expand the terms by applying distributive law & reorders the variable in the sum terms.
- ④ reduce the expression by omitting repeated sum terms in any, because $A \cdot A = A$.

Example - 1

Write the boolean Expression of minterms (SOP's) from the table

Inputs			Outputs
A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	1	1
1	1	0	0
1	1	1	1

Soln

Step ①

Note down input combinations for output 1

There are such cases 001, 011, 101, and 111

Step ②

write the product (AND) terms for each case.

A	B	C	Product
0	0	1	$\bar{A}\bar{B}C$
0	1	1	$\bar{A}BC$
1	0	1	$A\bar{B}C$
1	1	1	ABC

Step ③

$$\begin{aligned}\therefore \text{SOP} &= \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}C + ABC \\ &= m_1 + m_2 + m_3 + m_7\end{aligned}$$

The Expression can be further expressed as, $\text{SOP} =$

Use karnaugh map to simplify boolean Expression (upto 4 variables) in SOP-form.

Procedure to simplify SOP

- 1) Plot the k map and place 1's in those cells corresponding to 1's in the truth table. Others with 0
- 2) Circle the 1's who are not adjacent to any other.
- 3) make 8 grouping, 4-grouping, 2-grouping if possible.
- 4) Combine any pairs necessary to include any 1's that have not been grouped.
- 5) form the simplified Expression

2	6	14	10
---	---	----	----

$$Y = B\bar{C} + A\bar{C}D + \bar{A}\bar{B}C\bar{D}$$

$$\therefore \text{Expression } Y = B\bar{C} + A\bar{C}D + \bar{A}\bar{B}C\bar{D}$$

Use of k-map for POS form.

Same as SOP form but instead of ~~with~~ considering 1's we consider 0's output.

Minimize the Expression

$$Y = \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + AB\bar{C}\bar{D} + AB\bar{C}D + A\bar{B}\bar{C}D + \bar{A}B\bar{C}\bar{D}$$

$$Y = \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + AB\bar{C}\bar{D} + AB\bar{C}D + A\bar{B}\bar{C}D + \bar{A}B\bar{C}\bar{D}$$

0100 0101 0100 1101 1001 0010
 m_4 m_5 m_{12} m_{13} m_9 m_2

CD \ AB	00	01	11	10
00	0	1	1	0
01	0	1	1	1
11	0	0	0	0
10	1	0	0	0

$$Y = B\bar{C} + A\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D}$$

∴ Expression = $Y = B\bar{C} + A\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D}$

Use of k-map for POS form.

Same as SOP form but instead of ~~with~~ considering

1's we consider 0's output.

Example - 7

Reduce the following Equation using K-maps.

$$F(A, B, C, D) = \sum M(0, 2, 3, 8, 9, 12, 13, 15)$$

CD \ AB	00	01	11	10
00	0		12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

$$F = (\bar{A} + C)(\bar{C} + A + B)(D + \bar{A} + B)(\bar{A} + B + \bar{C} + D)(A + B + D)$$

$$F = (A + B + D)(\bar{A} + \bar{B} + \bar{D})(A + B + \bar{C})(\bar{A} + C)$$

Binary Codes

The process of converting numerical, alphabets & special characters into binary format is called "coding"

The combination of binary digits that represents numbers, alphabets & symbols called as "binary codes" codes are broadly classified into 5 groups.

- (i) weighted binary codes
- (ii) Non-weighted binary codes
- (iii) Error - detecting codes
- (iv) Error - correcting codes
- (v) Alphanumeric codes.

Weighted binary codes

In weighted codes, each digit's position of the number represents a specific weight.

Ex: If number 567 in decimal then 5 weights 100, 6 weights 10's & 7 weights 1's. $5 \times 100 + 6 \times 10 + 7 \times 1 = 567$ code - (BCD)

Un-weighted binary codes

Unweighted codes are not assigned with any weights to each digit position, each digit position within a number is not assigned fixed value.

Ex: Excess 3 codes, Gray codes.

Use of weighted codes

- ① Used for I/O operations in digital circuits
- ② Data manipulation during arithmetic operation.
- ③ To represent decimal digits in digital systems

Un-weighted codes

- ① Used in error detection & correction in communication systems
- ② Used for generating karnaugh maps.

BCD code [8421]

- ① BCD - Binary coded decimal
- ② It is a numerical code in which each digit of a decimal represented by a separate group of bits.
- ③ The most common BCD code is 8421 in which each decimal digit is represented by a 4-bit binary number.
- ④ It is a weighted code.
- ⑤ To express any decimal number in 8421 codes simply replace each decimal digit by 4 bit codes.

Parity Bit : parity bit is used to detect errors during transmission of binary information from one location to another

Parity Bit is an extra bit added with a binary message to make number of 1's either even or odd.

The circuit which generates the parity bit in transmitter is parity generator & the circuit which receives & checks parity in the receiver is called parity checker.

Chapters-2 Digital IC logic families

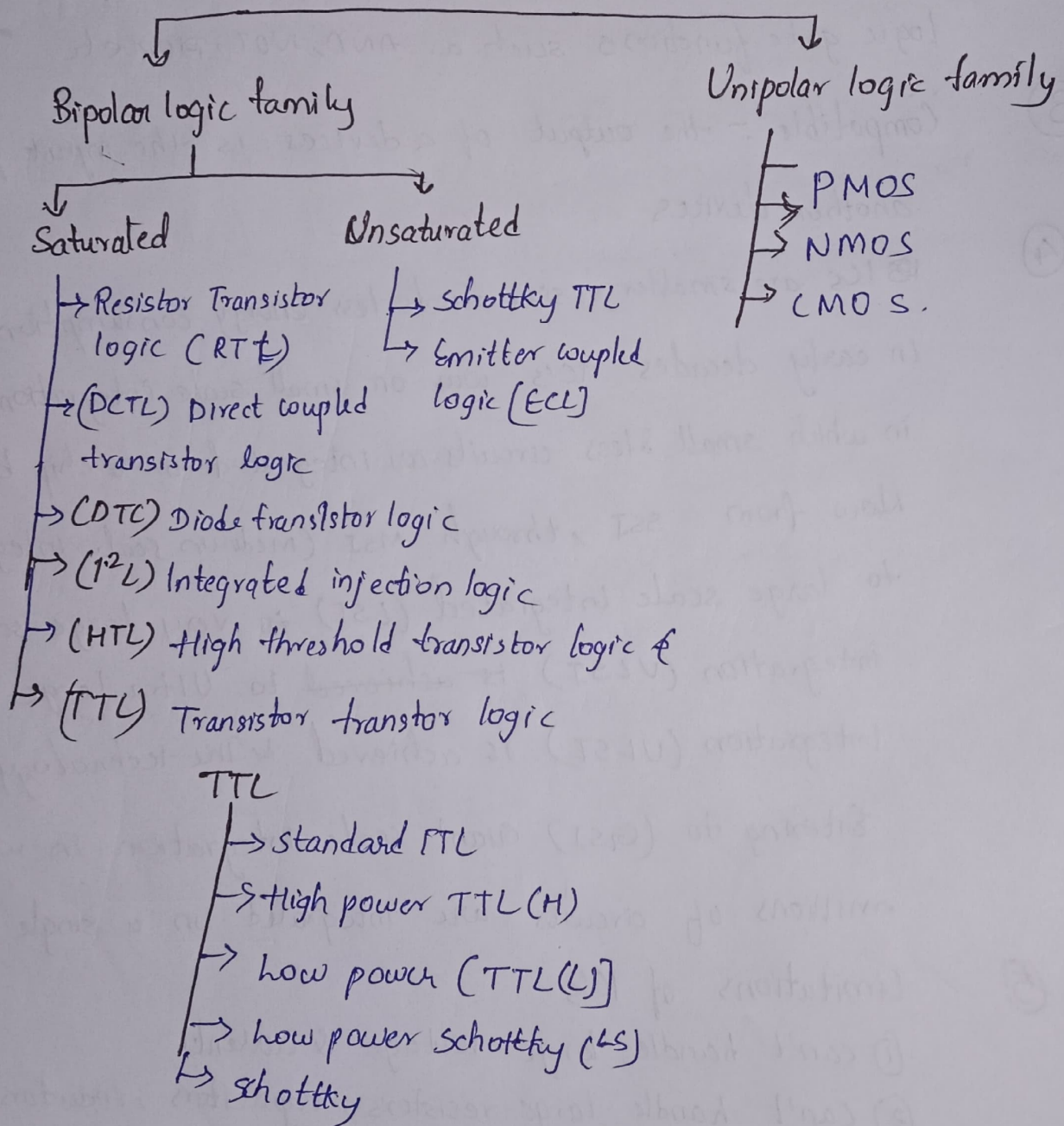
- ① Integrated circuit: Digital IC's are made using several different circuits & production technologies, on a single chip
- ② A Digital logic family: A digital logic family is a collection of different integrated circuits chips that are compatible devices and performs different logic gate functions such as AND, NOT, NOR etc
- ③ Compatible :- the output of a device is the input for another device
- ④ ICs are smaller in size needs less energy consumption
In early decades IC's was on small scale integration (SSI) in which small & less circuits are integrated on a chip but now from SSI, through MSI (medium scale integration) to large scale integrated (LSI) to very large scale integration (VLSI) is achieved to Ultra large scale integration (ULSI) is achieved & The technology is entering to (GSI) giant scale integration in which millions of circuits are integrated on a single chip.
- ⑤ Limitations of IC's
 - ① can't handle large voltage & currents
 - ② can't handle large resistors, capacitors, inductors, & transistors

Classification of logic families.

Digital logic family

ICs are designed using bipolar devices or metal oxide semiconductor devices or combination of both

Digital logic family



Fabrication: process of manufacture

Characteristics of Digital IC's :

- ① Input & Output logic levels
- ② Propagation delay
- ③ Noise margin
- ④ fan in
- ⑤ fan out
- ⑥ Power dissipation
- ⑦ Figure of merit
- ⑧ Power supply requirements

① Logic level of TTL

The values of voltages or current that are used to represent a LOW state & HIGH state is called logic level.

• In binary logic, 0 & 1 is used to represent these levels

Positive logic system: If higher voltage represents 1 & lower as 0 then it is positive logic system

Negative logic System: If lower voltage represent 1 & Higher as zero

Assume if +5V & 0V are logic level voltages then

Positive logic: High - 1 (5V) low - 0 (0V)

Negative logic: High - 0 (5V) low - 1 - (0V)

② Voltage requirements of TTL & CMOS

operating voltage: voltage at which device operates.

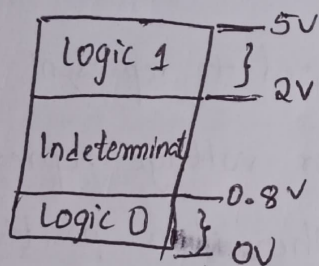
→ Different logic families require different operating voltage.

TTL requires +5V as operating voltage.

CMOS requires +3V & +15V

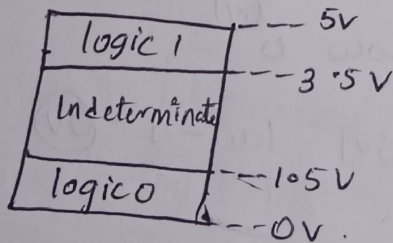
① For standard TTL devices a logic 0 ranges from 0 to 0.8V & logic 1 ranges from 2-5V.

② voltage between 0.8V to 2V are said to be indeterminate & should not be used as inputs to any TTL device



③ For CMOS voltage between 0 - 1.5V is logic 0 & voltage from 3.5 to 5V are defined for logic 1

④ Indeterminate range - 1.5V to 3.5V.



③ Propagation Delay

- * the maximum time taken by output to change its state in responding to change in input.
- * The speed of a digital circuit is specified in terms of propagation delay
- * Shorter the propagation delay, higher the speed of the circuit
- * Propagation delay (t_p) is measured using $t_p = \frac{t_{PHL} + t_{PLH}}{2}$
 - $t_{PHL} \rightarrow$ time taken to switch logic HIGH from LOW
 - $t_{PLH} \rightarrow$ time taken to switch logic LOW from HIGH.

④ NOISE ~~MARGIN~~

The unwanted voltages ^{called} noise may induce into circuit either internally or externally

This may cause the voltage logic levels to shift undesirable & results gets undesirable operation

NOISE immunity: The ability of a circuit to tolerate noise signals is called noise immunity

noise margin: The maximum noise voltage that can be added to an input signal so that it can't cause undesirable change in the circuit output is called noise margin.

* In other words, measure of noise immunity is called noise margin

⑤ Power dissipation

* Every IC requires certain amount of electrical power to operate. This power must not exceed limits.

* Hence, Power dissipation is the amount of power that can be taken for operation without destruction of IC.

* by $\rightarrow$ $P_D = V_{CC} \times I_C$

P_D = Power dissipation of the gate

V_{CC} = DC voltage

I_C = Average supply current

when power taken is beyond the limits then heat generated & destroys IC.

⑥ Figure of merit

Figure of merit of a digital IC is the product of speed & power.

Figure of merit = Propagation delay (ns) ^{nano seconds} $\times$ Power (mW)
measured in (PJ) - Pico joules.

Normally minimum value is desired.

⑦ FAN IN

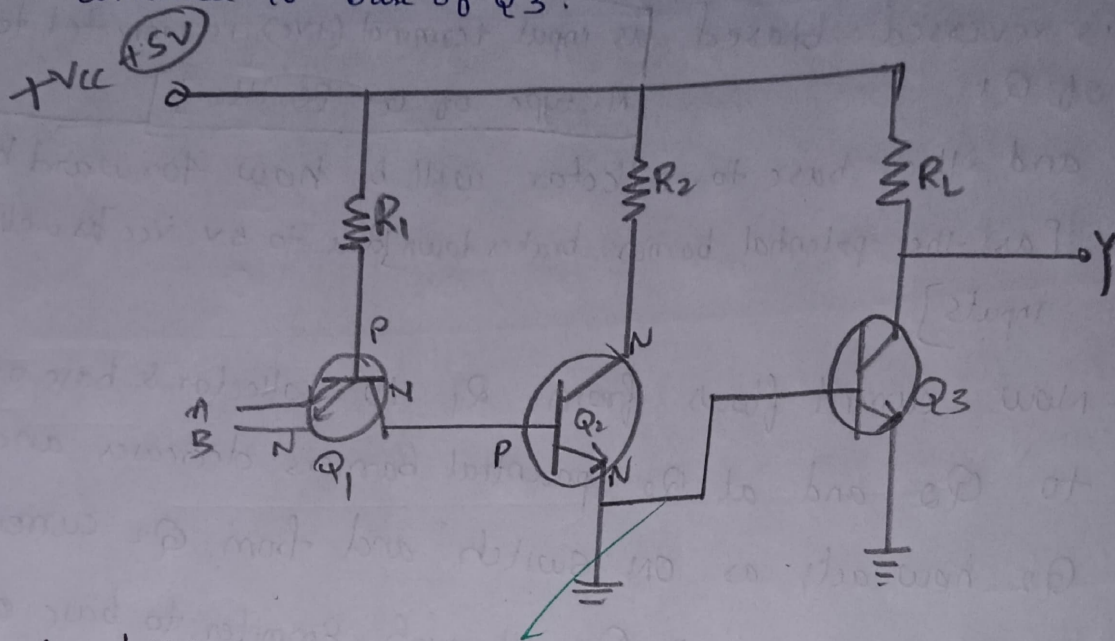
The maximum number of inputs that can be connected to logic circuits is called FAN-IN of the logic circuit.

Ex: Inverter has fan-in of 1 (-to-)

⑧ FAN OUT [loading factor]

The maximum number of outputs that can be driven by a logic circuit is called FAN-OUT.

- A two input TTL NAND gate is as follow
- Q_1 is multiple-emitter transistor for two inputs
- Q_1 collector is connected to base of Q_2 & Emitter of Q_2 is connected to base of Q_3 .



Working

- ① If A and B inputs are low [assume to be grounded] then the current due to V_{CC} flows through R_1 and then crosses Base to Emitter junction of Q_1 as A/B are grounded [low] hence Base Emitter is FB [as +ve terminal of V_{CC} is connected to P-type [Base]]
- ② The current cannot flow through Base to collector of Q_1 as collector of Q_1 is connected to Base of Q_2 i.e. reversed biased. [N to P] & V_{CC} +ve terminal of V_{CC} is connected to N-type collector of Q_2 it is also reversed biased hence connection of Q_2 is like OFF switch
- ③ Same with Q_3 as emitter of Q_2 is connected to base of Q_3 Q_3 is also OFF switch
- ④ Hence current from V_{CC} directly flows from R_L (Pull up resistor to output Y as no chance of grounding.
- ⑤ Even if one of the output may be High it can't pass through Q_2 as due to potential Barrier

Case-2

If A & B are HIGH

① If A & B are HIGH then emitter to base junction is reversed biased as input terminal (+ve) is connected to N-type of Q_1 [emitter]

② and the base to collector will be now forward biased [as the potential barrier breaks down] due to 5V V_{CC} as well as inputs]

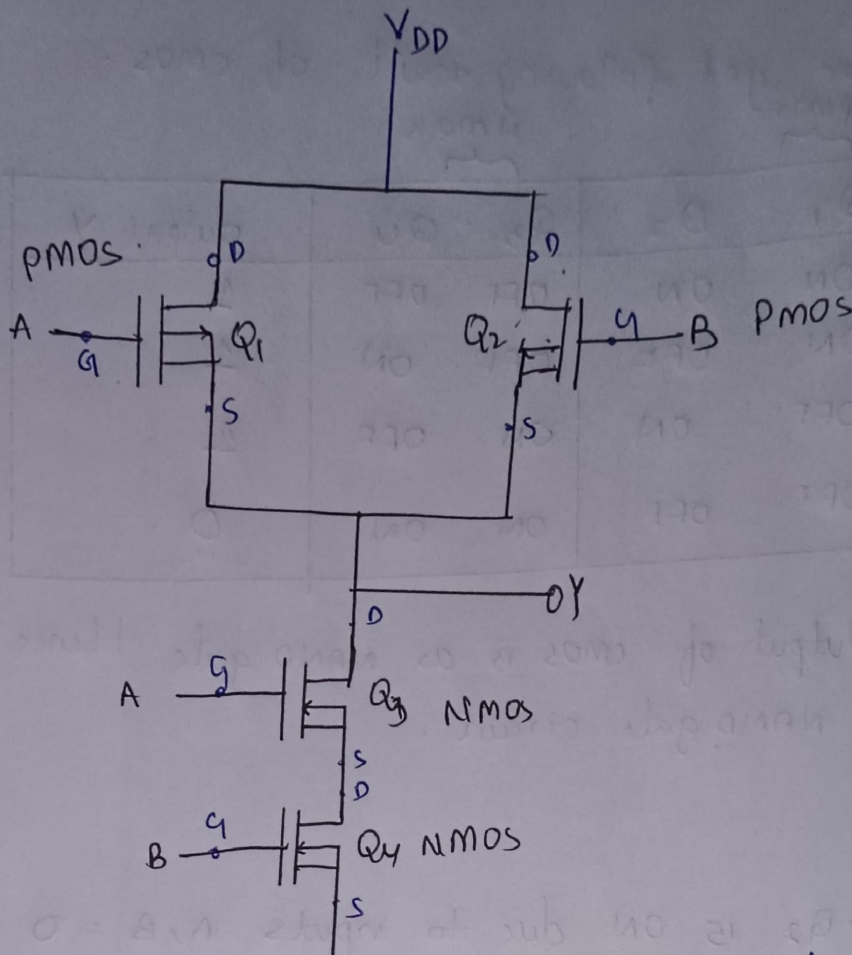
③ Now current flows from R_1 to collector & base of Q_1 to Q_2 and at Q_2 potential barriers decreases and Q_2 now acts as ON switch and from Q_2 current flows from Q_2 to Q_3 through emitter to base connection and at Q_3 it is grounded. & ON switch

④ The current from V_{CC} flows through R_2 and then is grounded due to Q_3 & output is zero.

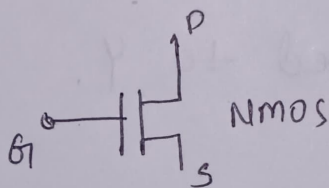
CMOS inverter

comparatively less fanin

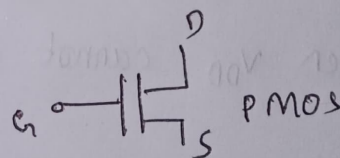
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- ① Q_1, Q_2 are PMOS and are connected in parallel.
- ② Q_3, Q_4 are NMOS and are connected in series & 1ed.
- ③ Transistors used here acts as switches.
- ④ At logic 0. Q_1, Q_2 (PMOS) Acts as ON and Q_3, Q_4 as OFF (NMOS)
- ⑤ At logic 1, Q_1, Q_2 (PMOS) Acts as OFF & Q_3, Q_4 (NMOS) Acts as ON.



$G=1 \rightarrow \text{ON}$
 $G=0 \rightarrow \text{OFF}$



$G=1 \rightarrow \text{OFF}$
 $G=0 \rightarrow \text{ON}$

⑥ O/p is taken b/w PMOS & NMOS